# Ph.D. Entrance Test – SYLLABUS

## 1.Research Methodology:

- Foundations of Research, Problem Identification & Formulation, Research Design, Qualitative and Quantitative Research, Measurement,
- Research Aptitude: Research meaning, ethics and characteristics, Type of Research, Methods of Research, and Thesis Writing: Its Characteristics & Format.
- Reasoning: Mathematical reasoning, Numerical reasoning, Arguments, deductive & inductive research, Logical & Venn diagram, Inferences, Analogies.
- Data Interpretation: Interpretation of data, Mapping analysis of data, Quantitative &qualitative research.
- Language Comprehension.

## 2.Mathematical Analysis:

**Real Number System:** Ordered sets, Fields, Real field, Extended real number system, Euclidean spaces. Basics of Set theory: Ordered pairs, Relation and functions, one-to-one correspondence, equivalent sets, cardinal number, finite and infinite sets, Countable and uncountable sets with examples.

**Basic Topology:** Metric spaces, Open sets, Closed sets, Compact sets, Perfect sets, Connected sets.

**Numerical Sequence and Series:** Convergent sequences, subsequences, Cauchy sequences, some special sequences. Series, Series of non-negative series, summation by parts, absolute convergence, addition and multiplication of series.

**Continuity:** Limits of function, Continuous function, Continuity and Compactness, Continuity and Connectedness, Discontinuity, Monotonic functions.

**Differentiation:** The derivative of a real function, Mean value theorems, the continuity of derivatives, Derivatives of higher order, Taylor's theorem, Differentiation of vector valued functions.

**The Riemann–Steiltje's Integral**: Definition and existence of the integral, Properties of the integral, Integration and Differentiation.

**Sequences and Series of Functions:** Pointwise and uniform convergence, Uniform convergence& continuity, Uniform convergence & integration, Uniform convergence & differentiation,

**Functions of Several Variables**: Linear transformations, invertible linear operators, matrix representation, Differentiation, partial derivatives, gradients, directional derivative, continuously differentiable functions, The contraction principle.

**The Inverse and Implicit Function Theorem:** The Inverse function theorem, Implicit function theorem with examples, Jacobians, Derivatives of Higher order and differentiation of integrals.

## 3. Measure Theory and Integration:

**Lebesgue Measure:** Introduction, Outer measure, measurable sets and Lebesgue measure, translation invariant, algebra of measurable sets, countable subadditivity, countable additively and continuity of measure, Borel sets, a non- measurable set.

**Measurable Function:** Examples: Characteristic function, constant function and continuous function, Sums, products and compositions, Sequential point wise limits, Simple functions, Littlewood's three principles.

**Lebesgue Integral of Bounded Functions:** The Riemann integral, integral of simple functions, integral of bounded functions over a set of finite measure, bounded convergence theorem.

**The General Lebesgue Integral:** Lebesgue integral of measurable nonnegative functions, Fatou's lemma, Monotone convergence theorem, the general Lebesgue integral, integrable functions, linearity and monotonicity of integration, additivity over the domains of integration. Lebesgue dominated convergence theorem.

**Differentiation and Integration:** Differentiation of monotone functions, Vitali covering lemma, Dini derivatives, Lebesgue differentiation theorem, functions of bounded variation, Jordan's theorem, differentiation of an integral, indefinite integral, absolute continuity.

## 4.Algebra:

**Groups:** Definition and examples of groups, Subgroups, abelian groups, cyclic groups, Lagrange's theorem, normal subgroups and quotient groups, homomorphism, isomorphism, Cauchy’s theorem for abelian groups, application of Cauchy’s theorem, automorphism, inner and outer automorphism.

**Permutation Groups:** Examples, orbit, cycle, transposition, alternating groups, Cayley’s Theorem, Conjugate class, class equation, Cauchy theorem for finite groups.

**Sylow’s Theorem and Problems:** solvable groups, direct products, Fundamental theorem on finite abelian groups.

**Rings:** Definition and examples of Rings, Integral domain, Field, Characteristic of a Ring, Homomorphism, Kernal, isomorphism, ideals and quotient rings, maximal ideal, prime ideal, principal ideal ring.

**Euclidean Ring:** Definition and examples, greatest common divisor, prime and irreducible elements, unique factorization domain, unique factorization theorem.

**Polynomial Rings:** Division Algorithm, irreducible polynomial, primitive polynomial, Gauss Lemma, Eisenstein criterion, polynomial ring over commutative rings.

**Extension Fields:** Definition and example, algebraic extension, transitivity of algebraic extension, roots of polynomial, Remainder Theorem, Factor theorem.

## 5. Linear Algebra:

**Vector spaces:** Definition and example, linear dependence and independence, Basis, dimension, subspaces, homomorphism, isomorphism, Hom(V, W) as a vector space, dual spaces.

**Inner Product Spaces:** Annihilator, Schwarz inequality, orthonormal basis, Gram-Schmidt orthogonalization process, orthogonal complement.

**Linear Transformations:** The algebra of Linear Transformation, singular and non singular transformations, characteristic polynomials, minimal polynomials, Rank and Nullity, Eigen values and eigen vectors.

**Matrix of Linear Transformation:** Examples, matrix of change of basis, similar matrices

**Canonical Forms**: Similar transformations, Invariant subspaces, Triangular forms, Nilpotent Transformations, Jordan form, Trace and Transpose, Determinants.

**Hermitian adjoint**: Hermitian transformations, Unitary and Normal Transformations, Real quadratic forms**:** Sylvester’s law of Inertia, rank and signature.

## 6.Differential Equations:

**First Order Linear Differential Equations:** Introduction, first order linear differential equations, separable equations, exact equations, Bernoulli's equation and method of substitutions.

**Higher Order Linear Differential Equations:** Homogeneous equations and general solutions; Initial value problems; existence and uniqueness of solutions, linear dependence and independence of solutions, Solutions of nonhomogeneous equations by Method of Variation of parameters, Method of Undetermined Coefficients. Homogeneous equation of order n, initial value problems, Non-homogeneous equations. Linear equations with variable coefficients, reduction of order of the equation.

**Oscillations of Second Order Equations:** Introduction, Oscillatory and non-Oscillatory differential equations and some theorems on it. Boundary value problems, Sturm Liouville theory; Green's function. Solution in Terms of Power Series: -Solution near an ordinary point and a regular singular point–Frobenius method–Legendre, Bessel's and Hypergeometric equations and their polynomial solutions, Rodregue's relation, generating functions, orthogonal properties, and recurrence relations.

**Successive Approximations Theory:** Introduction, solution by successive approximations, Lipschitz condition, Convergence of successive approximations, Existence and Uniqueness theorem (Picard's theorem).

**First Order Partial Differential Equations**: Introduction, Construction of First-order Partial Differential Equations, Solutions of First Order Partial Differential Equations, Solutions Using Charpit's Method, Method of Cauchy Characteristics, Method of Separation of Variables

**Second Order Partial Differential Equations:** Introduction, Origin of Second Order Equations, Equations with Variable Coefficients, Canonical Forms.

**Parabolic Equations:** Introduction, Solutions by Separation of Variables, Solutions by Eigenfunction Expansion Method, Solutions by Laplace Transform Method, Solutions by FourierTransforms Method, Duhamel's Principle, Higher Dimensional Equations, Solutions to parabolic equations in cylindrical and spherical coordinate systems.

**Hyperbolic Equations:** Introduction, Method of Characteristics (D'Alembert Solution), Solutions by Separation of Variables, Solutions by Eigenfunctions Expansion Method, Solutions by Laplace Transform Method, Solutions by Fourier Transform Method, Duhamel's Principle, Solutions to Higher Dimensional Equations, Solutions to hyperbolic equations in cylindrical and spherical coordinate systems.

**Elliptic Equations:** Introduction, Solutions by Separation of Variables, Solutions by Eigen functions Expansion Method, Solutions by Fourier Transform Method, Similarity Transformation Method, Solutions to Higher Dimensional Equations, Solutions to elliptic equations in cylindrical and spherical coordinate systems.

## 7.Complex Analysis:

**Analytic Functions**: Limits, continuity and differentiability of complex valued functions, Cauchy-Riemann equations, Laplace equation, harmonic functions, polynomials, Lucas's theorem.

**Power Series:** Sequence and series-review, uniform convergence, radius of convergence, powerseries as an analytic function, Abel's limit theorem.

**Conformal Mappings:** Arcs and closed curves, analytic functions in regions, principle of conformal mapping, length and area.

**Complex Integration:** Line integral, rectifiable arcs, line integrals as functions of arcs, Cauchy's theorem of rectangle, Cauchy-Goursat theorem, Cauchy's theorem ina disk.

**Cauchy's Integral Formula:** Index of a point with respect to a closed curve, the integral formula, representation formula.

**Higher Derivatives:** Morera's theorem, Liouville's theorem, fundamental theorem of algebra, Cauchy's estimate.

**Local Properties of Analytic Functions:** Isolated and non-isolated singularities, removable singularities, Taylors's theorem, zeros and poles, meromorphic functions, zeros and poles of order 'h', essential singularity, Weierstrass theorem.

**Maximum Modules Principle:** The maximum principle, Schwarz lemma, Some applications of Schwarz's lemma, Hadamard's three circles theorem.

**The General Form of Cauchy's Theorem**: Chains and cycles, general statement of Cauchy's theorem, locally exact differentials, multiply connected regions.

**Calculus of Residues:** Residue at a finite point, residue at the point at infinity, The Residue theorem, The argument principle, Rouche's theorem. Evaluation of the integrals of thetype, $\int_{\alpha}^{2\pi+\alpha} R(\cos\theta, \sin\theta)\, d\theta$, $\int_{-\infty}^{\infty} g(x) \cos mx\, dx$, Cauchy principal value.

**Harmonic Functions:** Laplace's equation, The Mean value property, maximum principle for Harmonic functions, Poisson's formula, Schwarz's formula, Schwarz's theorem, The reflection principle.

**Power Series Expansion:** Weierstrass's theorem, Hurwitz theorem, the Taylor series, the Laurent series.

## 8.Topology:

**Topological Spaces:** Basic topological spaces, Topological spaces: The definition and examples, Bases for a topology, Open sets and closed sets; Interior closure of a set, Exterior and boundary, Relative or subspace topology; sub-bases.

**Continuity and convergence**: Hausdorff spaces, continuous function, open and closed maps, Pasting Lemma, convergence, uniform convergence theorem, Homeomorphism, maps into products.

**Connectedness:** Connected spaces, path connected spaces; various counter examples, Components and path components; locally connected and locally path connected space.

**Compactness:** Converging properties, Lindelof spaces, Basic properties of Lindeloff compact spaces, Countable compactness; Sequential compactness, limit point compactness; Bolzano-Weierstrass property, Tychnoffs theorem.

**Countablity and Separation axioms:** First and second countable spaces, separable and Lindeloff spaces and examples.$T_0$, $T_1$and$T_2$ spaces, Hausdorff, Regular and completely regular spaces, normal and completely normal spaces; Complete and collection wise normal spaces; The countability axioms, Local compactness.

**Uryshon's Theorem:** Uryshon's Lemma Tietze's extension theorem, Uryshon's metrization theorem.

## 9.Numerical Analysis:

**Solutions of Linear System of Equations:** Introduction to Direct Methods via., Gauss Elimination method, Gauss-Jordan method, LU factorization, Triangularisation method, Iteration Methods: Gauss Jordan methods, Gauss-Seidel method, Successive Over relaxation method, Convergence Criteria, and problems on each method.

**Solutions of Nonlinear/Transcendental Equations:** Fixed point iteration, Method of Falsi position, Newton Raphson Method, Secant method, Regula-Falsi Method, Muller's Method, Aitken's $\Delta^2$ method, Orders of convergence of each methods. Problems on each method.

Origin of roots by Sturm Sequences. Extraction of quadratic polynomial by Bairstow's method. **Eigenvalues and Eigenvectors of A Matrix:** The characteristics of a polynomial, The eigenvalues and eigenvectors of matrix by Jocobi's method, Given's method, House holders method, power method, Inverse Power method, QR Algorithm.

**Interpolation Theory**: Polynomial interpolation theory, Gregory Newtons forward, back ward and Central difference interpolation polynomial. Lagranges interpolation polynomial, truncation error. Hermite interpolation polynomial, Inverse interpolation, Piece wise polynomial interpolation, Trigonometric interpolation, Convergence Analysis,

**Approximation Theory:** Introduction, Spline approximation, Cubic splines, Best approximation property, Least square approximation for both discrete data and for continuous functions.

**Numerical Differentiation and Integration:** Introduction, errors in numerical differentiation, Extrapolation methods, cubic spline method, differentiation formulae with function values, maximum and minimum values of a tabulated function, partial differentiation. Numerical Integration, Newton-Cotes integration methods; Trapezoidal rule, Simpson's $1/3^{rd}$ rule,

Simpson's $3/8^{th}$ rule and Weddle's rule. Gaussian integration methods and their error analysis. Gauss-Legendre, Gauss-Hermite, Gauss-Legurre and Gauss-Chebyshev integration methods and their error analysis. Romberg integration, Double integration.

**Numerical Solutions of Initial Value Problems (Ordinary Differential Equations):**

Introduction**,** Derivation of Taylor's series method**,** Euler's method, Modified Euler Method, Runge-Kutta Second, Third and Forth order methods, Runge-Kutta-Gill method, Predictor-Corrector methods; Milne's method, Adam's Bashforth Moulton method.

**Solutions of Boundary Value Problems (Ordinary Differential Equations):** Introduction, Solution of boundary value problems method of undetermined coefficients, Finite difference methods, Shooting Method, and Midpoint method.

**Numerical Solutions of Partial Differential Equations:** Introduction, Derivation of finite difference approximations to the derivatives, Solution of Laplace equation by Jacobi, Gauss Seidel and SOR Methods, ADI Method, Parabolic, Solution of heat equation by Schmidt and Crank-Nicolson Methods, Solution of wave equation using Finite difference method.

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4. Complex Analysis: ISBN-0-07-000657-1 : L.V. Ahlfors
5. Foundation of Complex Variables : S.Ponnuswamy
6. An Introduction to Ordinary Differential Equation : Eurl A. Coddington
7. Differential Equations with Applications and Historical Notes : Simmons, G.F.
8. Elements of Partial Differential Equations : I.N. Sneddon
9. Topics in Algebra: ISBN-9971-512-53-X : I.N.Herstein
10. Linear Algebra : Surjeet Sing
11. Linear Algebra : Hoffman and Kunze
12. A First Course in Topology: ISBN-81-203-2046-8 : J.R. Munkres
13. Numerical Analysis & Computation : E.K. Blum
14. Elements of Numerical Analysis : P. Henrici
15. Research Methodology: Methods and Techniques : C. R. Kothari
16. Research Methodology : D K Bhattacharyya

# Question Paper Pattern:

**PART-A**

1. **Twenty** Multiple Choice Questions.
2. Each question carries **1** mark.
3. Answer **ALL** the questions.

**PART-B**

1. **Eight** Short Answer Questions.
2. Each question carries **6** marks.
3. Answer any **FIVE** questions.

**PART-C**

1. **Six** Essay Type Questions.
2. Each question carries **10** marks.
3. Answer any **FOUR** questions.